# Package 'aspline' 

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Title Spline Regression with Adaptive Knot Selection
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Description Perform one-dimensional spline regression with automatic knot selection. This package uses a penalized approach to select the most relevant knots. B-splines of any degree can be fitted. More details in 'Goepp et al. (2018)', "'Spline Regression with Automatic Knot Selection", [arXiv:1808.01770](arXiv:1808.01770).

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aspline Fit B-splines with automatic knot selection.

## Description

Fit B-splines with automatic knot selection.

## Usage

aspline(
x ,
y,
knots $=\operatorname{seq}(\min (x), \max (x)$, length $=42)[-c(1,42)]$,
pen $=10^{\wedge} \operatorname{seq}(-3,3$, length $=100)$,
degree $=3 \mathrm{~L}$,
family = c("gaussian", "binomial", "poisson"),
maxiter $=1000$,
epsilon $=1 \mathrm{e}-05$,
verbose = FALSE,
tol $=1 \mathrm{e}-06$
)
aridge_solver(
x ,
$y$,

```
    knots = seq(min(x), max(x), length = 42)[-c(1, 42)],
    pen = 10^seq(-3, 3, length = 100),
    degree = 3L,
    family = c("gaussian", "binomial", "poisson"),
    maxiter = 1000,
    epsilon = 1e-05,
    verbose = FALSE,
    tol = 1e-06
)
```


## Arguments

| $x, y$ <br> knots <br> pen | Input data, numeric vectors of same length <br> Knots |
| :--- | :--- |
| degree | A vector of positive penalty values. The adaptive spline regression is performed <br> for every value of pen |
| family | The degree of the splines. Recommended value is 3, which corresponds to nat- <br> ural splines. |
|  | A description of the error distribution and link function to be used in the model. <br> The "gaussian", "binomial", and "poisson" families are currently implemented, <br> corresponding to the linear regression, logistic regression, and Poisson regres- <br> sion, respectively. |
| maxiter | Maximum number of iterations in the main loop. <br> epsilon |
| Value of the constant in the adaptive ridge procedure (see Frommlet, F., Nuel, G. <br> (2016) An Adaptive Ridge Procedure for LO Regularization.) |  |
| verbose | Whether to print details at each step of the iterative procedure. |
| tol | The tolerance chosen to diagnostic convergence of the adaptive ridge procedure. |

## Value

A list with the following elements:

- sel: list giving for each value of lambda the vector of the knot selection weights (a knot is selected if its weight is equal to 1.)
- knots_sel: list giving for each value of lambda the vector of selected knots.
- model: list giving for each value of lambda the fitted regression model.
- par: parameters of the models for each value of lambda.
- sel_mat: matrix of booleans whose columns indicate whether each knot is selected.
- aic, bic, and ebic: Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC), and Extended BIC (EBIC) scores, for each value of lambda.
- dim: number of selected knots for each value of lambda.
- loglik: log-likelihood of the selected model, for each value of lambda.


## Functions

- aridge_solver: Alias for aspline, for backwards compatibility.

```
bandsolve bandsolve
```


## Description

Main function to solve efficiently and quickly a symmetric bandlinear system. Theses systems are solved much faster than standards system, dropping from complexity $\mathrm{O}\left(\mathrm{n}^{3}\right)$ to $\mathrm{O}\left(0.5 * \mathrm{nk}^{2}\right)$, where k is the number of sub diagonal.

## Usage

bandsolve(A, b = NULL, inplace = FALSE)

## Arguments

A Band square matrix in rotated form. The rotated form can be obtained with the function as.rotated: it's the visual rotation by 90 degrees of the matrix, where subdiagonal are discarded.
b right hand side of the equation. Can be either a vector or a matrix. If not supplied, the function return the inverse of $A$.
inplace Should results overwrite pre-existing data? Default set to false.

## Value

Solution of the linear problem.

## Examples

```
A = diag(4)
A[2,3] = 2
A[3,2] = 2
R = mat2rot(A)
solve(A)
bandsolve(R)
set.seed(100)
n = 1000
D0 = rep (1.25, n)
D1 = rep(-0.5, n-1)
b = rnorm(n)
```


## Description

Create the penalty matrix

## Usage

band_weight(w, diff)

## Arguments

$\begin{array}{ll}w & \text { Vector of weights } \\ \text { diff } & \text { Order of the differences to be applied to the parameters. Must be a strictly }\end{array}$ positive integer

## Value

Weighted penalty matrix $D^{T} \operatorname{diag}(w) D$ where $\mathrm{D}<-\operatorname{diff}$ (diag(length(w) + diff), differences $=\operatorname{diff})$. Only the non-null superdiagonals of the weight matrix are returned, each column corresponding to a diagonal.
bladder
Bladder Cancer aCGH profile data

## Description

A dataset of 500 observations corresponding to 500 probes of the aCGH profile of a bladder cancer patient. The original data are provided by Stransky et al. (2006). This dataset consists of probes 1 through 500 of individual 1.

## Usage

bladder

## Format

A data frame with 500 observations and 2 variables:
$\mathbf{x}$ probe number
y aCGH profile value

## Source

Stransky, N., Vallot, C., Reyal, F., Bernard-Pierrot, I., de Medina, S. G. D., Segraves, R., de Rycke, Y., Elvin, P., Cassidy, A., Spraggon, C., Graham, A., Southgate, J., Asselain, B., Allory, Y., Abbou, C. C., Albertson, D. G., Thiery, J. P., Chopin, D. K., Pinkel, D. and Radvanyi, F. (2006). Regional Copy Number Independent Deregulation of Transcription in Cancer', Nature Genetics 38(12), 1386-1396.
block_design Transform a Spline Design Matrix in block compressed form

## Description

Transform a Spline Design Matrix in block compressed form

## Usage

block_design(X, degree)

## Arguments

$\begin{array}{ll}X & \text { The design matrix, as given by splines2: :bSpline. } \\ \text { degree } & \text { Degree of the spline regression, as used in function splines2: :bSpline. }\end{array}$

## Value

A matrix $B$ with all non-zero entries of $X$ and a vector of indices alpha representing the positions of the non-zero blocks of $X$.

```
coal Yearly number of coal mine disasters in Britain
```


## Description

A data of 112 observations registering the yearly number of coal mine disasters in Britain from 1851 to 1962. The data comes from Diggle et al. (1988) and has been used for spline regression by Eilers et al. (1996).

## Usage

coal

## Format

A data frame with 112 observations and 2 variables:
year year
n number of coal mine disasters

## Source

Diggle, P. and Marron, J. S. (1988). 'Equivalence of Smoothing Parameter Selectors in Density and Intensity Estimation', Journal of the American Statistical Association 83(403), 793-800.

## References

Eilers, P. H. C. and Marx, B. D. (1996). 'Flexible Smoothing with B-splines and Penalties', Statistical Science 11(2), 89-102.

```
fossil Fossil data
```


## Description

A dataset with 106 observations on fossil shells from the SemiPar package (https://CRAN.R-project. org/package=SemiPar).

## Usage

fossil

## Format

A data frame with 106 observations and 2 variables:
age The age of fossils, in millions of years
strontium.ratio Ratio of strontium isotopes ...

## Source

Bralower, T.T, Fullagar, P.D., Paull, C.K, Dwyer, G.S. and Leckie, R.M. (1997). Mid-cretaceous strontium-isotope stratigraphy of deep-sea sections. Geological Society of America Bulletin, 109, 1421-1442.

## References

Ruppert, D., Wand, M.P. and Carroll, R.J. (2003). Semiparametric Regression, Cambridge University Press.
helmet Testing Crash Helmets

## Description

A dataset containing the acceleration and time after impact of helmets from a simulated motorcycle accident.

## Usage

helmet

## Format

A data frame with 132 rows and 2 variables:
$\mathbf{x}$ Time after impact, in milliseconds
y Head acceleration, in units of $g \ldots$

## Source

Dataset number 338 of Hand, D. et al. (1993) A Handbook of Small Datasets.

```
hessian_solver Inverse the hessian and multiply it by the score
```


## Description

Inverse the hessian and multiply it by the score

## Usage

hessian_solver(par, XX_band, Xy, pen, w, diff)

## Arguments

par
XX_band

Xy The vector of currently estimated points $X^{T} y$, where $y$ is the y-coordinate of the data.
pen Positive penalty constant.
w Vector of weights. Has to be of length
diff The order of the differences of the parameter. Equals degree +1 in adaptive spline regression.

## Value

The solution of the linear system:

$$
\left(X^{T} X+\operatorname{pen} D^{T} \operatorname{diag}(w) D\right)^{-1} X^{T} y-\operatorname{par}
$$

LDL LDL

## Description

Fast inplace LDL decomposition of symmetric band matrix of length $k$.

## Arguments

D Rotated row-wised matrix of dimensions $\mathrm{n} * \mathrm{k}$, with first column corresponding to the diagonal, the second to the first super-diagonal and so on.

## Value

List with D as solution of our LDL decomposition.

## Examples

```
n=10;
D0=1:10;
D1=exp(-c(1:9));
D=cbind(D0, c(D1,0))
sol=LDL(D)
```

lidar Lidar data

## Description

Data from a light detection and ranging (LIDAR) experiment

## Usage

lidar

## Format

range distance travelled before the light is reflected back to its source
logratio logarithm of the ratio of received light from two laser sources

## Source

- Sigrist, M. (Ed.) (1994). Air Monitoring by Spectroscopic Techniques (Chemical Analysis Series, vol. 197). New York: Wiley
- The R package https://CRAN.R-project.org/package=SemiPar


## References

Ruppert, D., Wand, M.P. and Carroll, R.J. (2003). Semiparametric Regression, Cambridge University Press.

$$
\text { mat2rot } \quad \text { Rotate a band matrix to get the rotated row-wised matrix associated. }
$$

## Description

Rotate a symmetric band matrix to get the rotated matrix associated. Each column of the rotated matrix correspond to a diagonal. The first column is the main diagonal, the second one is the upper-diagonal and so on. Artificial 0 are placed at the end of each column if necessary.

## Usage

mat2rot(M)

## Arguments

M
Band square matrix or a list of diagonal.

## Value

Rotated matrix.

## Examples

```
A = diag(4)
A[2,3] = 2
A[3,2] = 2
## Original Matrix
A
## Rotated version
R = mat2rot(A)
R
rot2mat(mat2rot(A))
```

```
montreal Montreal Temperature Data
```


## Description

A dataset containing the tempature in Montreal for two years

## Usage

montreal

## Format

A data frame with 730 rows and 2 variables:
day The day of the year from January 1, 1961, to December 31, 1962
temp Temperature in Celsius ...

## Source

```
    fda::"MontrealTemp"
```

nmr Nuclear Magnetic Resonance data

## Description

A signal of nuclear magnetic resonance.

## Usage

nmr

## Format

Data farme of 1024 rows and two columns: the index $x$ and the signal $y$.

## Source

- Data from https://web.stanford.edu/~hastie/ElemStatLearn/datasets/nmr1.csv.
- See also The Elements of Statisical Learning (2001, 2nd Ed.), Hastie, T., Friedman, J., and Tibshirani, R.J, p. 176
rot2mat Get back a symmetric square matrix based on his rotated row-wised version.


## Description

Get back a symmetric square matrix based on his rotated row-wised version. The rotated form of the input is such each column correspond to a diagonal, where the first column is the main diagonal and next ones are the upper/lower-diagonal. To match dimension, last element of these columns are discarded.

## Usage

$$
\operatorname{rot} 2 m a t(R)
$$

## Arguments

R Rotated matrix.

## Value

Band square matrix.

## Examples

```
D0 = 1:5;
D1 = c(0,1,0,0);
D2 = rep(2,3);
A = rot2mat(cbind(D0,c(D1,0),c(D2,0,0)))
A
mat2rot(rot2mat(cbind(D0,c(D1,0),c(D2,0,0))))
```

    titanium Titanium heat data
    
## Description

A data set of 49 samples expressing the thermal property of titanium

## Usage

titanium

## Format

49 observations and two variables:
$\mathbf{x}$ temperature
y physical property

## Source

- de Boor, C., and Rice, J. R. (1986), Least-squares cubic spline approximation. II: variable knots. Report CSD TR 21, Purdue U., Lafayette, IN.
- Dierckx, P. (1993), Curve and Surface Fitting with Splines, Springer.
- Jupp, D. L. B. (1975), Approximation to data by splines with free knots, SIAM Journal on Numerical Analysis, 15: 328-343.

weight_design_band | Fast computation of weighted design matrix for generalized linear |
| :--- |
| model |

## Description

Fast computation of weighted design matrix for generalized linear model

## Usage

weight_design_band(w, alpha, B)

## Arguments

w Vector of weights.
alpha Vector of indexes representing the start of blocks of the design matrix, as given by block_design.
B Design matrix in compressed block format, as given by block_design.

## Value

Weighted design matrix $X^{T} \operatorname{diag}(w) X$ where X is the design matrix and $\mathrm{W}=\operatorname{diag}(\mathrm{w})$ is a diagonal matrix of weights.

```
wridge_solver
Fit B-Splines with weighted penalization over differences of parameters
```


## Description

Fit B-Splines with weighted penalization over differences of parameters

## Usage

```
wridge_solver(
    XX_band,
    Xy,
    degree,
    pen,
    w = rep(1, nrow(XX_band) - degree - 1),
    old_par = rep(1, nrow(XX_band)),
    maxiter = 1000,
    tol = 1e-08
)
```


## Arguments

XX_band The matrix $X^{T} X$ where X is the design matrix. This argument is given in the form of a band matrix, i.e., successive columns represent superdiagonals.
Xy The vector of currently estimated points $X^{T} y$, where y is the y-coordinate of the data.
degree The degree of the B-splines.
pen Positive penalty constant.
w Vector of weights. The case $\mathbf{w}=\mathbf{1}$ corresponds to fitting P-splines with difference \#' order degree + 1 (see Eilers, P., Marx, B. (1996) Flexible smoothing with $B$-splines and penalties.)
old_par Initial parameter to serve as starting point of the iterating process.
maxiter Maximum number of Newton-Raphson iterations to be computed.
tol The tolerance chosen to diagnostic convergence of the adaptive ridge procedure.

## Value

The estimated parameter of the spline regression.

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