Package ‘Pade’

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Type Package

Title Padé Approximant Coefficients

Version 1.0.6

Date 2023-10-12

Description Given a vector of Taylor series coefficients of sufficient length as input, the function returns the numerator and denominator coefficients for the Padé approximant of appropriate order (Baker, 1975) <ISBN:9780120748556>.

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Imports utils

Suggests covr, tinytest

URL https://github.com/aadler/Pade

BugReports https://github.com/aadler/Pade/issues

Encoding UTF-8

NeedsCompilation no

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`Pade-package` *Padé Approximant Coefficients*

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Details

The DESCRIPTION file:

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Author(s)

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Pade *Padé Approximant Coefficients*

Description

Given Taylor series coefficients a_n from $n = 0$ up to $n = T$, the function will calculate the Padé $[L/M]$ approximant coefficients so long as $L + M \leq T$.

Usage

```
Pade(L, M, A)
```

Arguments

L	Order of Padé numerator
M	Order of Padé denominator
A	vector of Taylor series coefficients, starting at x^0

Details

As the Taylor series expansion is the "best" polynomial approximation to a function, the Padé approximants are the "best" rational function approximations to the original function. The Padé approximant often has a wider radius of convergence than the corresponding Taylor series, and can even converge where the Taylor series does not. This makes it very suitable for computer-based numerical analysis.

The $[L/M]$ Padé approximant to a Taylor series $A(x)$ is the quotient

$$\frac{P_L(x)}{Q_M(x)}$$

where $P_L(x)$ is of order L and $Q_M(x)$ is of order M. In this case:

$$A(x) - \frac{P_L(x)}{Q_M(x)} = \mathcal{O}\left(x^{L+M+1}\right)$$

When q_0 is defined to be 1, there is a unique solution to the system of linear equations which can be used to calculate the coefficients.

The function accepts a vector `A` of length `T + 1`, composed of the a_n of the of truncated Taylor series

$$A(x) = \sum_{j=0}^{T} a_j x^j$$

and returns a list of two elements, `Px` and `Qx`, the Padé numerator and denominator coefficients respectively, as long as $L + M \leq T$.

Value

Pade returns a list with two entries:

Px Coefficients of the numerator polynomial starting at x^0.

Qx Coefficients of the denominator polynomial starting at x^0.

Author(s)

Avraham Adler <Avraham.Adler@gmail.com>

References

Baker, George Allen (1975) *Essentials of Padé Approximants* Academic Press. ISBN 978-0-120-74855-6

See Also

This package provides similar functionality to the pade function in the pracma package. However, it does not allow computation of coefficients beyond the supplied Taylor coefficients and it expects its input and provides its output in ascending—instead of descending—order.

Examples

```
A <- 1 / factorial(seq_len(11L) - 1) ## Taylor sequence for e^x up to x^{10} around x_0 = 0
Z <- Pade(5, 5, A)
print(Z)                              ## Padé approximant of order [5 / 5]
x <- -.01                             ## Test value
Actual <- exp(x)                      ## Proper value
print(Actual, digits = 16)
Estimate <- sum(Z[[1L]] * x ^ (seq_along(Z[[1L]]) - 1)) /
    sum(Z[[2L]] * x ^ (seq_along(Z[[2L]]) - 1))
print(Estimate, digits = 16)          ## Approximant value
all.equal(Actual, Estimate)
```

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