An introduction to the R package sn

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Skewed world of data:
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Note: this document refers to 'sn' version 1.5-0

Scheme

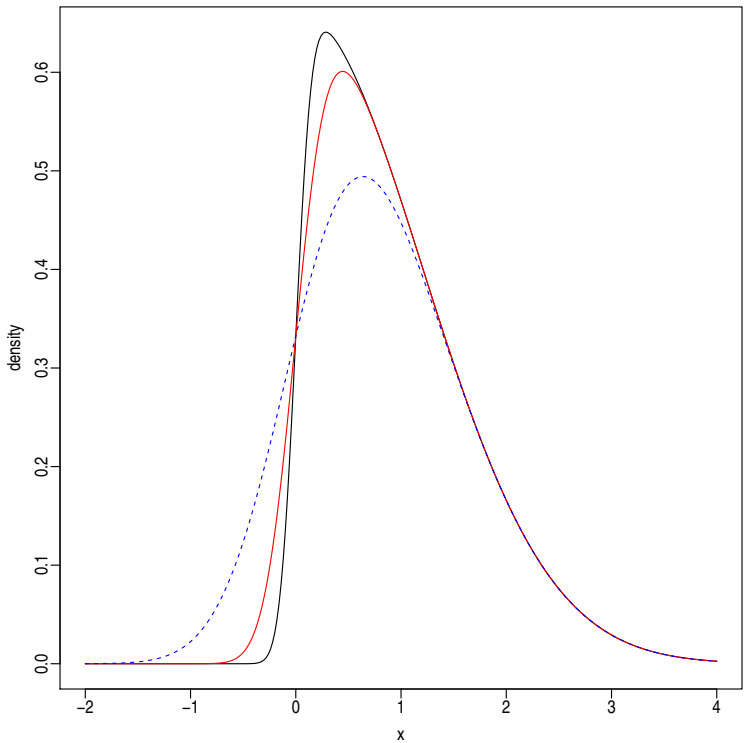
Pacakge `sn`: a one-slide panorama

- probability
  - classical-type functions for distributions:
    - univariate dist'n: `{d,p,q,r}{sn,st,sc}`
    - multivariate dist'n: `{d,p,r}{msn,mst,msc}`
    - utilities: `dp2cp`, `cp2dp`, `{sn,st}.cumulants`,...
    - build your own dist'n: `{d,r}{SymmModulated}`, `{d,r}{mSymmModulated}` and plot (if $d = 2$)
  - SEC distribution 'objects'
    two classes: `SECdistrUv`, `SECdistrMv`; protocol: S4
    - create object: `makeSECdistr` (also `extractSECdistr`)
    - manipulate it: `marginalSECdistr`, `affineTransSECdistr`, `conditionalSECdistr` (only for SN)
    - methods: `summary`, `plot`, `show`, `mean`, `sd`, `var`, ...
- statistics
  - core general function: `selm` (two object types: `selm`, `mselm`)
  - methods: `show`, `plot`, `summary`, `coef`, `residuals`, `fitted`, `predict`, `logLik`, `profile`, `confint`, ...

Prob-std

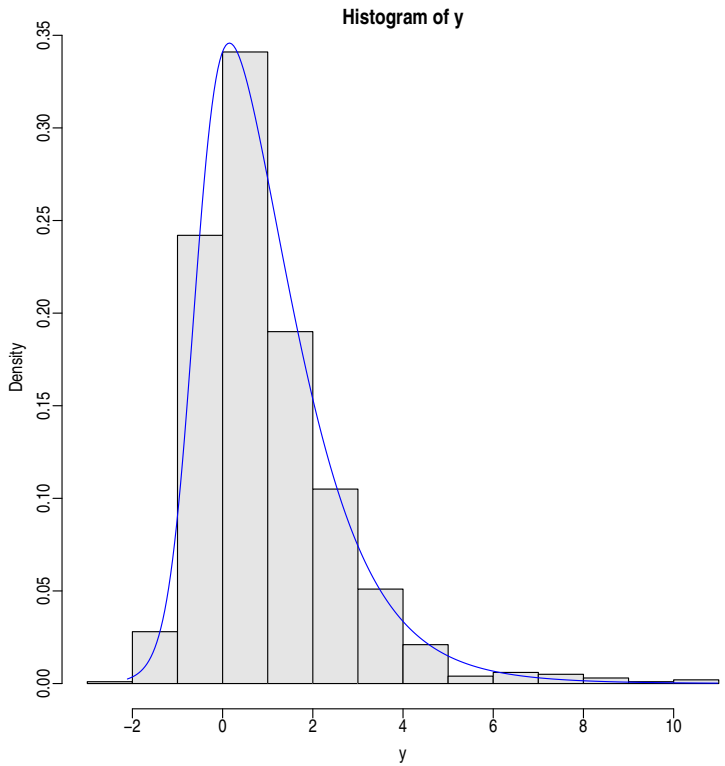
Simple functions: dsn and plotting SN density

```
# plotting SN densities
library(sn)
x <- seq(-2, 4, length=301)
y1 <- dsn(x, xi=0, omega=1.2, alpha=10)
plot(x, y1, type="l", ylab="density")
y2 <- dsn(x, 0, 1.2, 5)
lines(x, y2, col=2)
y3 <- dsn(x, dp=c(0, 1.2, 2))
lines(x, y3, col=4, lty=2)
```



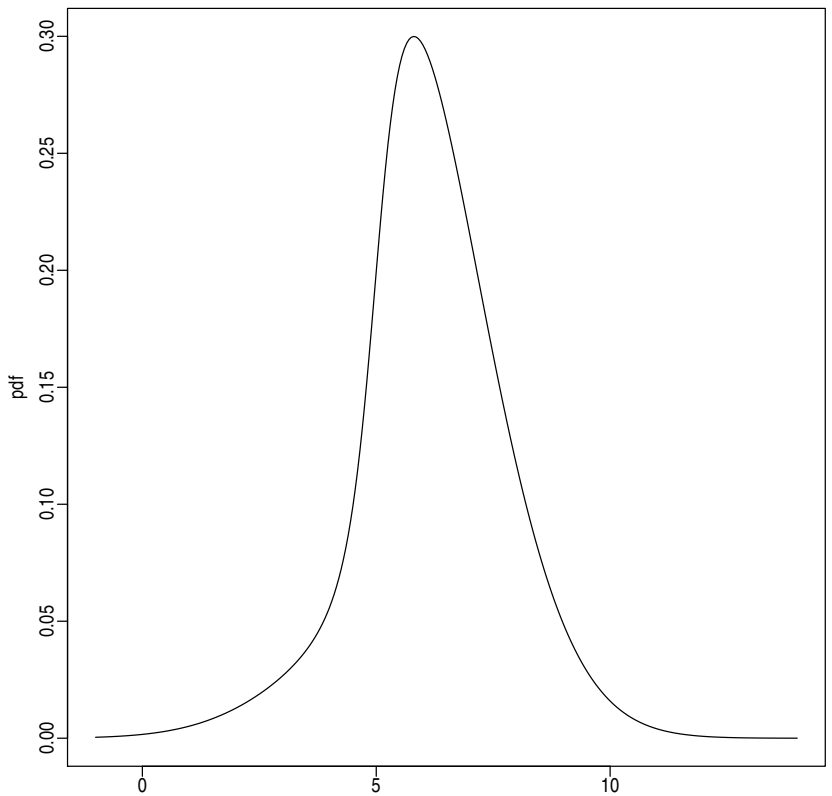
Simple functions: cp2dp, dp2dp, rst, dst

```
# switch between parameterizations
cpST <- c(1, 1.5, 1.5, 5.1)
# CP = (mean, st.dev, gamma1, gamma2)
dpST <- cp2dp(cpST, family="ST")
print(dpST)
#      xi   omega   alpha      nu
# -0.6202  1.8754  3.6733  7.1797
print(dp2cp(dpST, family="ST"))
# back to cpST
#
# sampling from ST and plotting density
y <- rst(1000, dp=dpST)
hist(y, prob=TRUE, col="gray90")
x <- seq(min(y), max(y), length=301)
pdfST <- dst(x, dp=dpST)
lines(x, pdfST, col=4)
print(sd(y)) # about cpST[2]
```



Unleash your creativity: case $d = 1$

```
# SGN of Arellano-Valle et al. (2004)
wSGN <- function(z, lambda)  z * lambda[1]/sqrt(1 + lambda[2]*z^2)
x    <- seq(-1, 14, length=301)
pdf  <- dSymmModulated(x, 5, 2, f0="normal", G0="normal", w=wSGN,
           lambda=c(3,5))
plot(x, pdf, type="l")
```



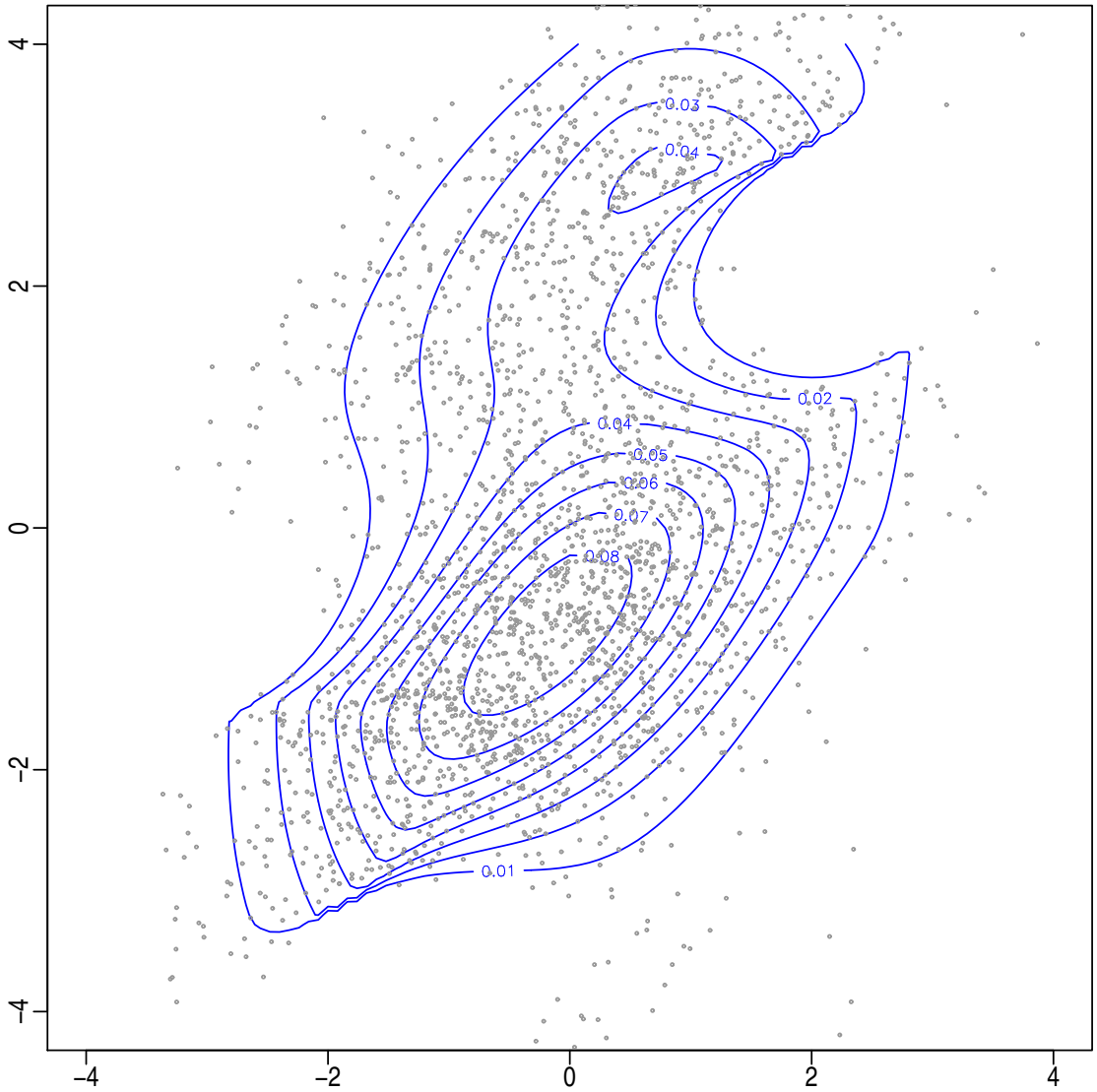
now change f0, G0, w,... and

make your density

Unlash your creativity: case $d > 1$

```
x <- matrix((1:12)/3, 4, 3)
S <- diag(1:3) + outer(1:3,1:3)/2
wMvTrigs <- function(z, p, q) sin(z %*% p)/(1 + cos(z %*% q))
pdf <- dmSymmModulated(x, xi=1:3, Omega=S, f0="t", G0="logistic",
       w=wMvTrigs, par.f0=5, par.G0=NULL, p=c(2,3,-2), q=c(1,1, 0))

# plotting when d=2
range <- cbind(c(-4,4), c(-4,4))
plot2D.SymmModulated(range, xi=c(0,0), Omega=S[1:2,1:2],
       f0="normal", G0="normal", w=wMvTrigs,
       par.f0=NULL, par.G0=NULL, p=c(1,-3), q=c(1,1), col=4)
y <- rmSymmModulated(2500, xi=c(0,0), Omega=S[1:2,1:2],
       f0="normal", G0="normal", w=wMvTrigs,
       par.f0=NULL, par.G0=NULL, p=c(1,-3), q=c(1,1))
points(y, cex=0.3, col="gray60")
```

Unlash your creativity: plotting if $d = 2$



Prob-obj

Working with distribution 'objects'

- Idea is to make a SEC distribution and work with this object
- Once the distribution object is created, we can:
  - extract/compute various characteristics
  - plot it
  - manipulate it to create a new distribution (if $d > 1$)
- Technical note: distributions are S4-type objects

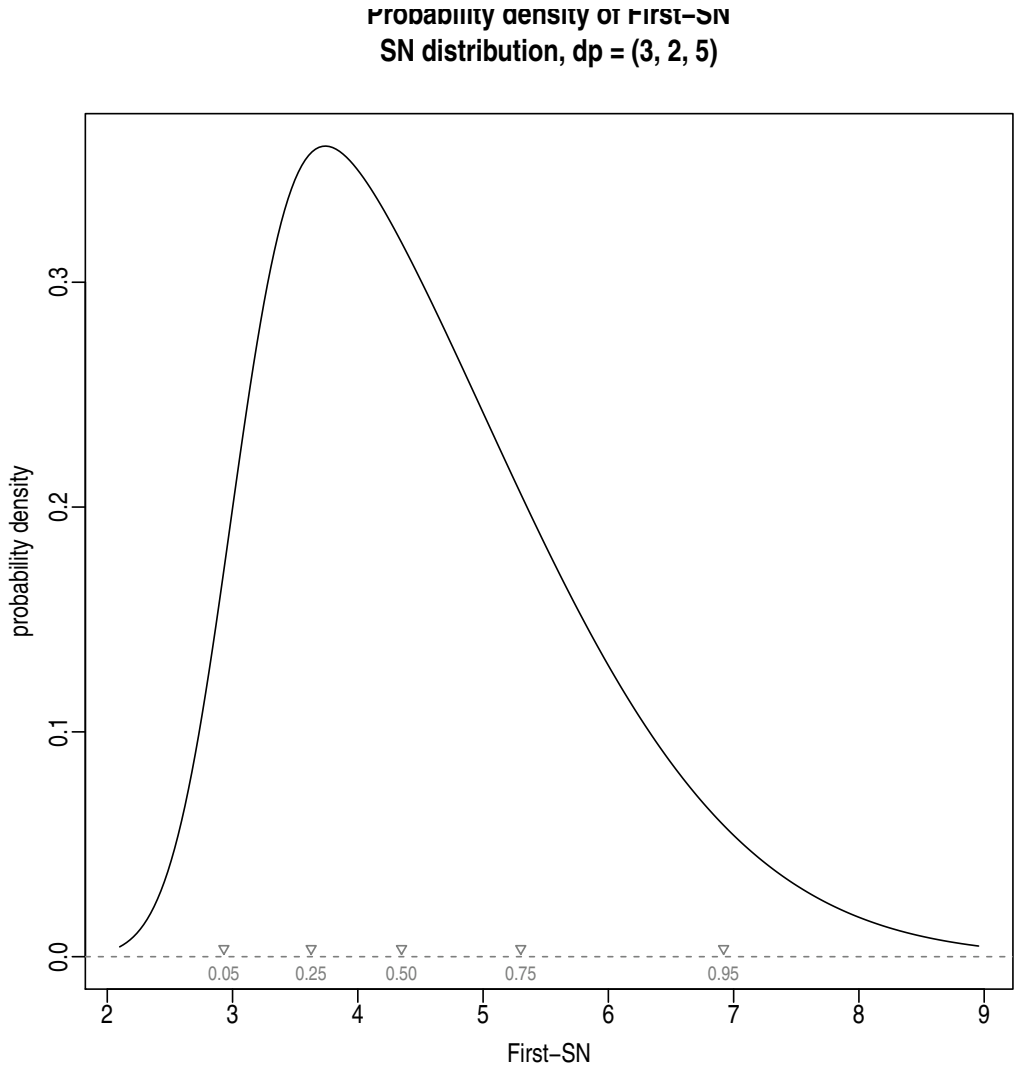
How to create a distribution

- Chief procedure is `makeSECdistr`
  (another route is `extractSECdistr` from `selm` or `mselm` object)
- we must to specify DP parameters
  - if univariate case, DP is assigned as a vector
  - if multivariate case, DP is assigned as a list
- we must to specify the `family`,
  options are: `"SN"`, `"ESN"`, `"ST"`, `"SC"`
- optional: name of the distribution, names of components
- `?makeSECdistr` tells you everything

A simple illustration

```
f1 <- makeSECdistr(dp=c(3,2,5), family="SN", name="First-SN")
#
show(f1) # of just type 'f1'
# Probability distribution of variable 'First-SN'
# Skew-elliptically contoured distribution of univariate family SN
# Direct parameters:
#   xi omega alpha
#    3     2     5

summary(f1) # longer output
#
c(mean(f1), sd(f1)) # lends 4.565 1.246

plot(f1)

# many possible variants, such as:
plot(f1, probs=c(0.1, 0.9))

?plot.SECdistr # says more, see method for signature 'SECdistrUv'
```

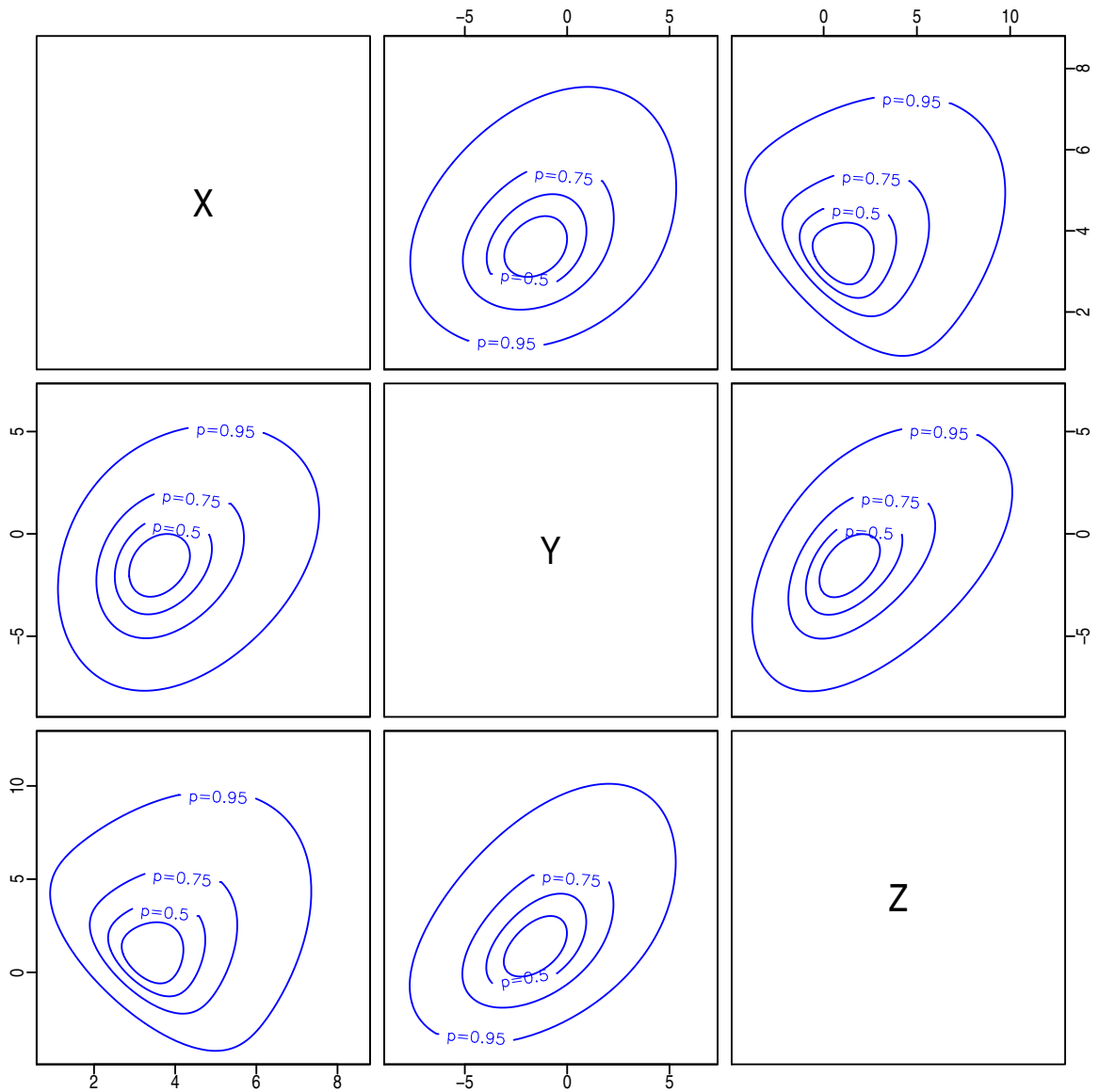
A simple illustration – plot 'First-SN'



An illustration with $d = 3$

```
dp3 <- list(xi = c(3, -2, 0),  Omega = diag(1:3) + outer(1:3,1:3)/2,
          alpha = c(5, -2, 6), nu = 5)
st3 <- makeSECdistr(dp=dp3, family="ST", name="Multiv.ST",
          compNames=c("X", "Y", "Z"))
show(st3) # of just type 'st3'

mean(st3)
#     X      Y      Z
# 3.944 -1.253  2.194

vcov(st3) # 3x3 variance matrix

summary(st3) # longer output

plot(st3, col="blue", landmarks="", main=NULL)  # note p=0.xx labels

?plot.SECdistr # look at method for signature 'SECdistrMv'
```

An illustration with $d = 3$: matrix plot



Manipulation of a distribution

- Refers to multivariate distributions only
- `affineTransSECdistr(object, a, A, <etc>)`
  applies affine transformation $a + A'Y$,
  where a is m-vector and A is $d \times m$ matrix
- `marginalSECdistr(object, comp, <etc>)`
  get marginal distribution of `comp` components from `object`
- `conditionalSECdistr(object, fixed.comp, fixed.values, <etc>)`
  applies conditioning on `fixed.comp` components
  (only for `"SN"` and `"ESN"` distributions)

Stats

Function selm for model fitting

Naming: lm + <Skew-Elliptical error> = selm

Also similar logic of lm: fit a linear model to location parameter

fit <- selm(response ~ formula , family ="SN" , <plus>)

- fit: S4 obj
- response: vector, matrix
- formula: like in lm
- family ="SN": also "ST", "SC"

Optional <plus> terms include:

data, subset the same as in lm

start starting values

method estimation methods are "MLE" and "MPLE";
the latter can be used to set a prior distribution of α

penalty only relevant for method="MPLE"

fixed.param allows only limited specifications, such as nu=<value>
(if family="ST") and alpha=0

... additional options

Methods for a `selm|mselm` object

classes `selm` returns a S4 object of class `selm` (for univariate response) or `mselm` (for multivariate response)

methods for each class, a bounch of 'methods' are available

- methods like for `lm` S3-objects: summary, plot, residuals, fitted, coef, predict, confint
- additional methods: logLik, profile, vcov

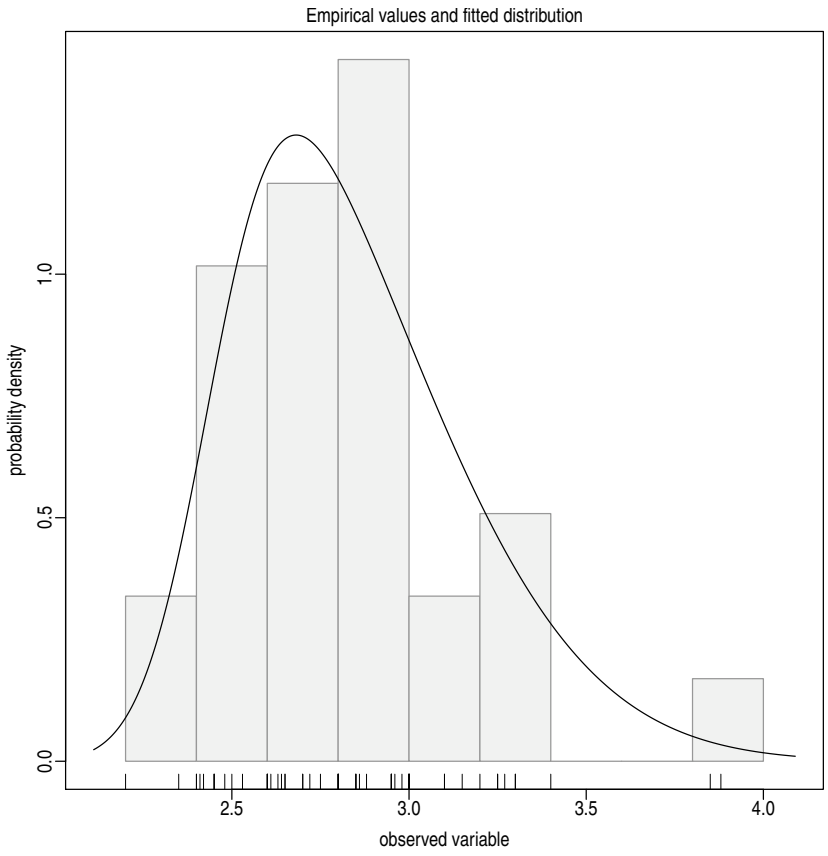
in addition extractSECdistr supplies a link to the probability section

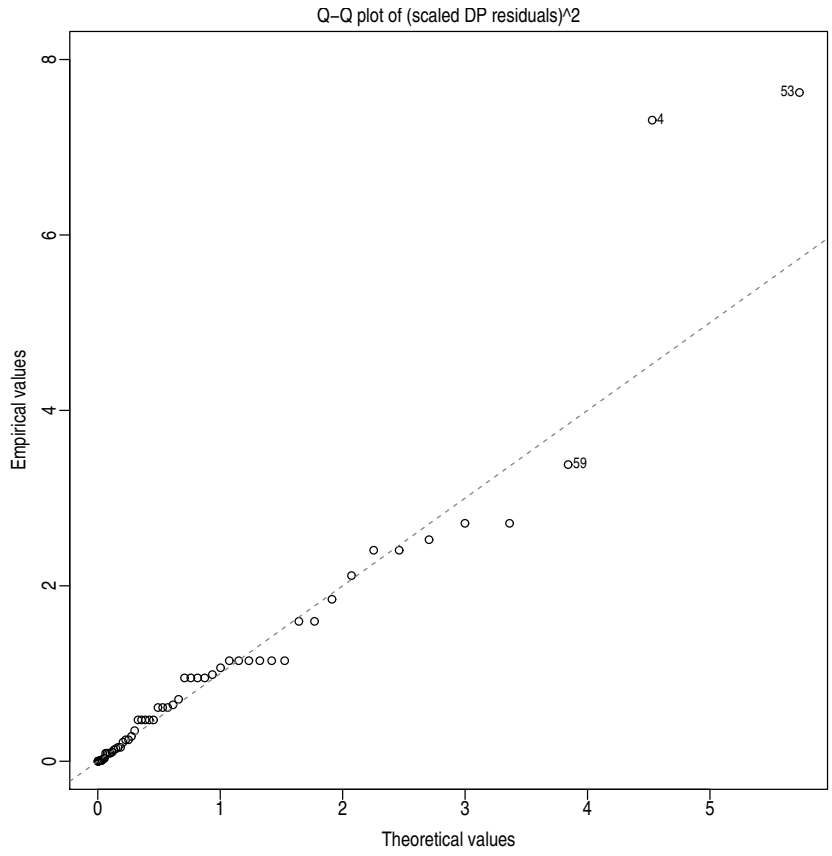
Note: for simpler interpretability, the default parameterization is CP. This contrasts with probability section which uses DP.

A simple example: Barolo phenols

```
library(sn)
data(wines)
olo.ph <- wines[wines$wine=="Barolo", "phenols"]
fit <- selm(olo.ph ~ 1, family="SN")
plot(fit, which=2:3)
#
# try
summary(fit)   # works with CP
summary(fit, param.type="DP")
```

A simple example: Barolo phenols – plots

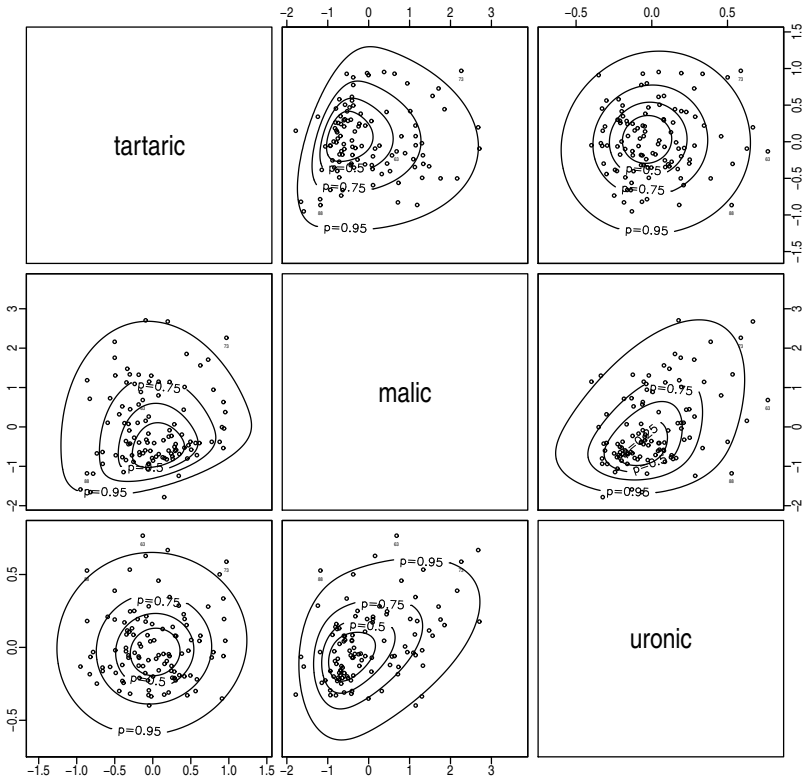


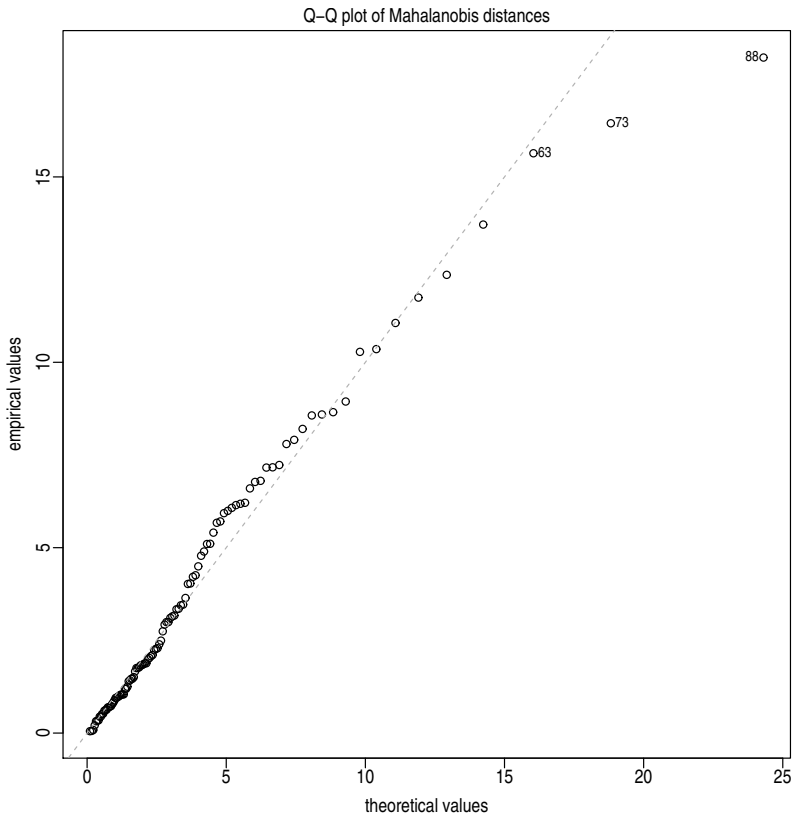


A multivariate example: más vino

```
library(sn)
data(wines)
fit2 <- selm(cbind(tartaric, malic, uronic) ~ colour + hue,
             family="ST", data=wines, subset=(wine != "Grignolino"))

plot(fit2, which=2:3)

summary(fit2)  # works with CP
summary(fit2, param.type="DP")

# constraint on degrees of freedom:
fit3 <- selm(cbind(tartaric, malic, uronic) ~ colour + hue,
            family="ST",  fixed.param=list(nu=8),
            data=wines, subset=(wine != "Grignolino"))
```

A multivariate example: más vino, plots





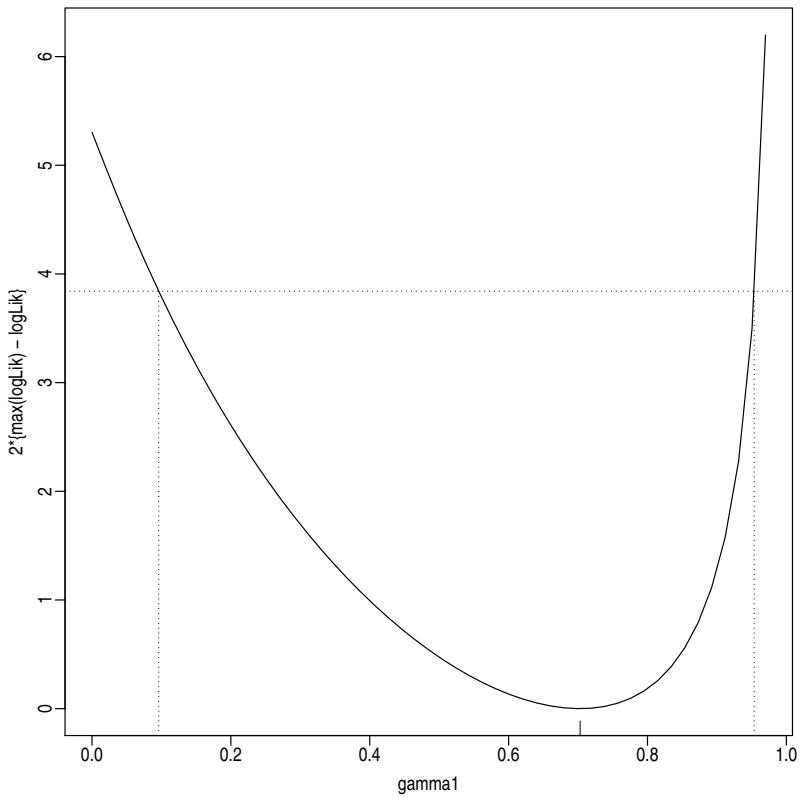
For the more adventurous: profile log L (LRT in fact)

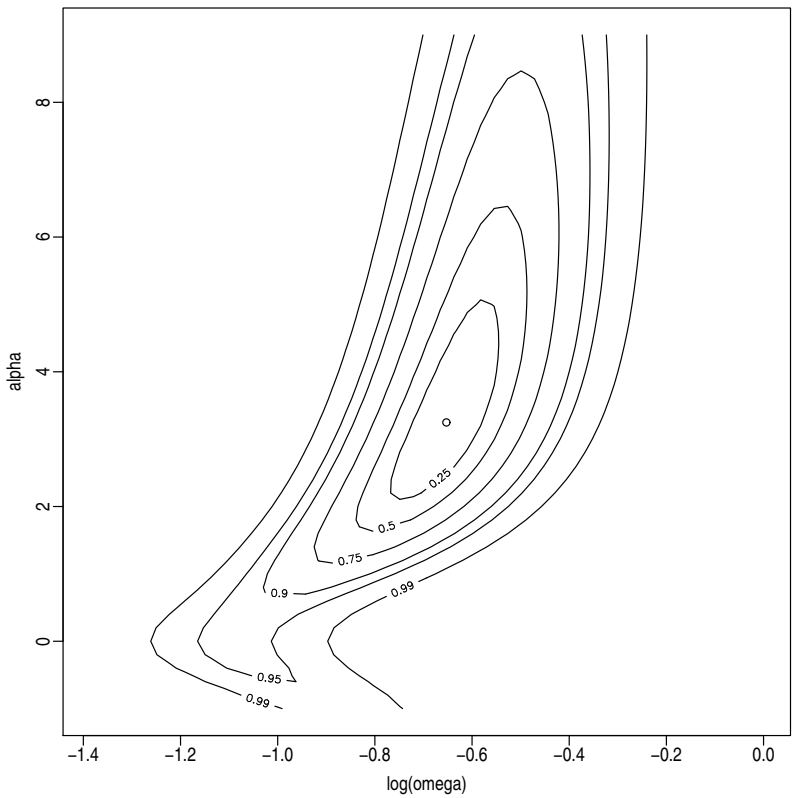
```
# re-use earlier model for Barolo phenols
show(fit)
# Object class: selm
# Call: selm(formula = ba.ph ~ 1, family = "SN")
# [...omissis...]

summary(fit)
# [...omissis...]
# Parameters of the SEC random component
#         estimate std.err
# s.d.       0.337    0.04
# gamma1     0.703    0.26

pll <- profile(fit, "cp", param.name="gamma1", param.val=c(0, 0.97))

profile(fit, "dp", param.name=c("omega", "alpha"),
        param.val=list(c(0.25, 1), c(-1, 9)),  npt=c(51,51) )
```

Note: works for `selm`-class objects, not `mselm`-class

For the more adventurous: profile log L, plots

Plots of (profile) Deviance $\equiv$ LRT statistic





Only for the more technically oriented people

- `selm` is the user interface function
- `selm` prepares work for the lower-level function `selm.fit`
- however, not even `selm.fit` performs the actual fitting
- depending on the fitted model, specific functions are called: `sn.mple`, `st.mple`, `msn.mle`, `msm.mple`, `mst.mple`
- To improve efficiency, one can call `selm.fit` directly, at che cost of some more programming effort
- One can even call the bottom-level functions, below `selm.fit`
- For more details, see `?selm.fit`

q()

q()